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Modern Algorithmic Aesthetics and Mathematical Harmony Throughout History

Nessreen Y. Ibrahim*

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Abstract

The main objective of this review is to emphasize the importance of mathematics and its role in aesthetic mental cognition and to encourage designers and contemporary artists to study pure mathematics seriously rather than dealing with pre-generated codes. By cultivating a philosophical attitude to so-called Algorithmic Aesthetics, the paper assumed that a new era of pure aesthetics could be begun. The article drew a solid connection between aesthetics and algorithmic design thinking based on mathematical approaches. Issues reviewed and discussed include the theoretical (quantitative) and applied (mathematical) theories that had been produced to explore the origin of aesthetics. This review attempts to emphasize the importance of mathematical sciences in understanding the beauty and perceive aesthetic phenomena. This connection drew on the precedents; Ceno-Pythagorean categories, Peircean universal categories, Birkhoff's aesthetic measurement, Claude Shannon's information theory, Moles' communicative system, Bense's information aesthetics, Siegfried Maser's cybernetic model, and Stakhov's mathematical harmony theory, which all contributed to reconceiving the concept of mathematical harmony in the aesthetic phenomenon. In line with these theories, algorithmic generative experiments were designed based on a naturally inspired algorithm, to present an example for a coherent mathematical structure that is required for creating any aesthetic phenomenon.

Keywords: Algorithmic aesthetics, Mathematical harmony, Information design, Generative art

Introduction

In humanity's journey, there was a will to discover the truth of life, the laws that control the existences, and to search the absolutes to reach the core of living. Philosophers, physicists, astronomers, chemists, engineers, architects, artists, and mathematicians were searching, studying, and analyzing nature and beyond. In the early days, the knowledge was genetic enough that mathematicians were studying the real world. Nowadays the world became more and more complex, where mathematics and natural sciences go different ways, and many mathematicians had decided to limit their activity to the problems of pure mathematics (Stakhov, 2009). The American mathematician Morris Kline in his book “Mathematics: The Loss of Certainty” mentioned that the researchers should return to nature and natural sciences, which were the original objectives of mathematics (Kline, 1980). But through history, there was the resistance of the efforts to that attempt to apply scientific methods and mathematics to Aesthetics. Mathematics as an abstract language that had been created or discovered to understand the physical world was always far from evaluating aesthetic phenomena. There were some efforts to make a bridge between Mathematics and Art, but these were just fuzzy attempts. However, in the late 1950's scientific works shown...
the ability to apply scientific procedures to understand aesthetics and art (Berleant, 1967). But that was just an action to understand how to produce art and how to critic. George David Birkhoff [1884–1944] the mathematician who was interested in his early life in music, formalized new structural aspects of aesthetic perception.

Where, he emphasizes the importance of pattern and tones that plays a great role in aesthetic perception (Douchová, 2016). While discussing his teleology theory (Huntley, 1970), claims that beauty has existed and esthetic has been evaluated through time history for a certain purpose. As hunger and thirst ensure our bodily survival, while sex drives for the survival of the race, and fear has survival value, the question here is for what purpose esthetic faculty comes for? Why human needs these subliminal emotions? Psychologists describe emotions as activities of the unconscious mind, therefore aesthetic experience is the resuscitation of subliminal emotions, and beauty is the power to evoke these emotions. So, the question could be more holistic; whether the mathematical nature of reality is mental; constructed by our minds or it is the abstract formula that controls the universe? Evaluating aesthetics was always a real subject to understand how feelings, happiness, emotional existence form and generate in humans.

In this paper, the concept of evaluated quantitative aesthetics, algorithmic aesthetics, the formulae for aesthetic measurements, the basic assumptions and applications of mathematical harmony are defined and discussed with a concentration on two main aspects; the development of Numerical Harmony in history as a theoretical Approach and the development of Mathematical Aesthetics as an applied mathematical approach, by reviewing different types of concepts used in measurement, the used mathematical notations, evaluation methods, and their applications.

Numerical Harmony (Theoretical Approach)

Beauty Through Mathematics in History

In his “Metaphysics | 1st century BC”, the first major work in the history of philosophy (Cohen et al., 2020), Aristotle [384–322 BC] summarize the harmony theory by mentioning the Pythagorean numerical harmony:
“The so-called Pythagoreans, studying mathematical sciences, for the first time have moved them forward and basing on them, began to consider mathematics as the beginnings of all things ... Because all things became like to numbers, and numbers occupied first place in all nature, they assumed that the elements of numbers are the beginning of all things, and that all universe is harmony and number.” (Aristotle, 1933).

Pythagoreans were the first to address the concept of mathematical harmony and its existence in the universe. Plato [about 428–348 BC] in the early times has pointed to the beauty lays in nature. His shocking statement about artworks as an imitation of imitation was just an appreciation for the laws and rules behind the visible world. He mentioned the importance of these laws and rules in his ‘Philebus | 4th century BC’:

“If arithmetic, mensuration, and weighing be taken away from any art, that which remains will not be much”, and again: “For measure and proportion always pass into beauty and excellence” (Birkhoff, 1933).

While Aristotle, saw more clearly the relation existing between beauty and mathematical formulas:

“Those are mistaken who affirm that the mathematical sciences say nothing of beauty or goodness .... The main elements of beauty are order, symmetry, definite limitation, and these are the chief properties that the mathematical sciences draw attention to”. (Aristotle, 1933).

In the seventeenth and early eighteenth century, a clear understanding of mathematical harmony has been arisen that led to recognize aesthetics as numerical relations and proportions; where the mathematician Pacioli [1447–1517] in his “De divina proportione | 1509” considers the golden ratio to be fundamentally important, and his close friend Michelangelo [1475–1564] ascribed the ideal human figure to the golden proportions. Also, the mathematician Leibnitz [1646 – 1716], who follows a rationalistic philosophy employed earlier by Descartes [1596 – 1650] describes music as the number of certain orderly relations among the notes, estimated intuitively, measures the aesthetic effect by “... counting performed by the mind without knowing that it is counting” (Birkhoff, 1933). The musician Jean-Philippe Rameau [1683-1764] in his “Traité de l’harmonie | 1722”, and “Démonstration du principe de l’harmonie |1751” claims that harmony is the basis for the melody Fig. 1, which is the basis of pure nature. According to his philosophy music is an “Imitation of Nature” and to develop a pleasant harmony, or a coherent melody, arithmetic relationships have to be applied so that music produced consonances (Marques, 2012).

Relating the philosophical aspects of beauty and aesthetics to computational approach was first made by the mathematician George David Birkhoff [1884–1944] as a mathematical formula [mentioned in detail below in section (2.1)], that estimate a measurable aesthetic value by drawing a ratio between “Order” which represents the number of regularities, and “Complexity” which is the number of elements in an image (Pascoal, 2015). Fig. 2.

One of the simplest definitions of beauty that summarized Birkhoff’s theorem was S. T. Coleridge’s definition; “beauty is the unity in variety”. J. Bronowski said in-that:

“Science is nothing more than the search to discover unity in the wild variety of nature or more exactly, in the variety of our experience” (Bronowski, 1956).

G. H. Hardy in his ‘A Mathematician’s Apology essay’ claimed that “I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our creations, are simply our notes of our observations.” (Hardy and Cain, 1940).

From Aristotelian Categories to Quantitative Aesthetic

Aesthetics, rather than any other field of knowledge, has been reluctant to apply any kind of scientific methods or rational evaluation techniques. Despite attempts that date back over three centuries ago which tried to quantitate aesthetics, the concerted aim of calculating and quantifying aesthetics is seemingly elusive and ineffable (Berleant, 1967). However, Aristotelian Realism was an important contribution that influenced many later aestheticians perceiving that the
surrounding objects could be quantified and interpreted mathematically. Aristotle attempts to classify things that existed in the world into general entities that were addressed according to their general kinds. He listed ten categories of things: Substance (with two subcategories primary substance and secondary substances), Quantity, Quality, Relation, Place, Date, Posture, State, Action, Passion (Thomasson, 2019).

Immanuel Kant [1724-1804], in his “Critique of Pure Reason | 1781/1787 ”, listed his categories by addressing the forms of objective empirical judgments. Kant begins from Aristotelian logic by distinguishing twelve pure concepts divided into four classes of three; Quantity (Unity, Plurality, Totality); Quality (Reality, Negation, Limitation); Relation (Inherence and Subsistence, Causality and Dependence, Community); Modality (Possibility, Existence, Necessity) (Thomasson, 2019).

While Aristotle had postulated ten categories and Kant twelve's, Peircean universal categories, which were created by Charles Sanders Peirce [1839–1914], presented his categorization theorem as a phenomenon of a triadic relation that systematically described the objects as; Firstness, Secondness, and Thirdness relations, coherently underpin heuristic principles that address all processes of perception, imagination, reasoning, and expression (Mittelberg, 2019), in which Firstness is the mode of being that is positive and addresses the category of potentiality, freedom, the immediacy of undifferentiated quality, and independence. Secondness involves the relation of a first to the second category of comparison, facticity, action, reality, and experience in time and space, and finally, Thirdness brings a second with a third that is a category of mediation, habit, memory, continuity, synthesis, communication, representation, and signs (Nöth, 1995).

In the 1800s, Birkhoff's theory was the first attempt to interpret beauty by numeral notation. A numeral notation is a writing system for expressing numbers that use digits/symbols to represent numbers of a given set that was developed to address positional numeral systems that are usually based on a certain extension. such as the Roman numerical system which presents the digit with only one value, or the Babylonian numeral system which the base was 60. Hindu–Arabic numeral system which we use in our classical mathematics based on ten, while the binary numeral system which used in computers based on two (Stakhov, 2009). Y. Mitropolsky in his preface of Alexey Stakhov's “The Mathematics of Harmony” book emphasized the importance of Mathematics of Harmony as a gap filler between Elementary Mathematics which is the basis of modern mathematical education and modern Advanced Mathematics. Mathematics as a discipline was developed out from four periods; “Mathematics Origin” which begins with the ancient civilizations; “Elementary Mathematics” from 6-5th centuries BC to 17th century; “Higher Mathematics” which was the beginning of using differential & integral calculus, and analytical geometry; and finally, “Modern Mathematics”

<table>
<thead>
<tr>
<th>Universal Categories</th>
<th>Semiotic relations</th>
<th>Correlates of triadic sign relations</th>
<th>1st trichotomy (presentative)</th>
<th>2nd trichotomy (representative)</th>
<th>3rd trichotomy (interpretative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firstness</td>
<td>possibility (feeling)</td>
<td>Similarity</td>
<td>Representamen</td>
<td>Qualisign</td>
<td>Icon</td>
</tr>
<tr>
<td>Secondness</td>
<td>actuality, facts (acting)</td>
<td>Contiguity</td>
<td>Object</td>
<td>Sinsign</td>
<td>Index</td>
</tr>
<tr>
<td>Thirdness</td>
<td>law, mediation (thinking)</td>
<td>Conventionality</td>
<td>Interpretant</td>
<td>Legisign</td>
<td>Symbol</td>
</tr>
</tbody>
</table>

which Lobachevsky's imaginary geometry is considered the beginning of this period. These stages of mathematical development through the written history were driven by two main problems; count problem, which came with the development of oral and written symbols to solve simple arithmetic notations, and measurement problem, which emphasizes the importance of understanding the physical approach of the surroundings such as irrigation systems inland areas, directions through astronomy, the quantity of grain, length of roads, etc... (Štakhov, 2009). Number theory and Measurement theory became the basis of "Classical Mathematics."

In “Introduction to informational aesthetics | 1969", Max Bense [1910–1990] interprets the Peircean universal categories and his verbal given conceptions to quantitative and mathematical formulas. The Peircean triadic sign relation consisting of M or medium relation, O or object relation, and I or interpretant relation Table 1 (Walther, 2000). Peirce's visualization of the classifications of Signs, and Bense's contributions [as shown below] were milestones in transforming Aristotelian and Ceno-Pythagorean categories theoretical approach to quantitative categories with mathematical approach.

Information Theory and Informative Aesthetics

The late 1950s was the era of room-size mainframe computers that extends through the 1960s' transistor-operated and mini-computers. At the end of the 1970s, the era of micro-computers had begun. This was the desktop computer era that paved the road for today's ubiquitous digital devices (Higgins and Kahn, 2012). Synchronously, in 1958 a mathematical theory of communication was influenced by cybernetics, which had some abilities in applying scientific procedures to study aesthetic perception. In the early 1960s, the concept of “Numerical Harmony” popped out again, and because of the information and computational revolution, the concept of Information Aesthetics has come into use as a computer-related approach (Nake, 2012).

Abraham Moles [1920–1992] and his book “Information Theory and Esthetic Perception | 1973” gave developed evidence of aesthetic informative perception. The theory deals with aesthetics as information that is a measurable quantity, independent of the particular message considered, and susceptible to statistical treatment (Berleant, 1967). Abraham A. Moles distinguishes between so-called Semantic Information and Aesthetic Information, Fig. 3. The semantic information is the message contained in sequences of logical, structured, expressible, and translatable data, while aesthetic information considered the
uncertainties, preferential, sensorial, qualitative data which are untranslatable.

Max Bense described the artwork as a phenomenon that pushes the boundaries and expands the experience of the viewer. He claimed that the balance between creativity and convention is art’s raison d’être. This balance could be automatically produced, statistically measured, and mathematically defined by doing methodical investigations of the possibilities and limits of creativity within a system.

Bense’s theory Fig. 4 of generative art assumes not only that aesthetic experience can be measured but that it can also be rationally and predictably produced (Caplan, 2020).

We live in a network of visible and invisible functions and relations, structures, and aggregates made of artificial phenomena and synthetic elements (Bense, 1949).

Alexander Gottlieb Baumgarten [1714-1762] mentioned that the mind’s logical and sensual cognition is equally important (Nake, 2012). Abstracts, concepts, and semiotics are considered elements in value judgment and are not avoided. We have to understand the way that our mind generates thoughts, feelings, and cognition in the context of bio-informative signals that are produced by an aesthetic communication process.

Siegfried Maser [1938–2016] in his book “Numerische Ästhetik | 1970” assumed that the main objective of aesthetic information is to measure the objective values of any object by measuring general features contingent with the physical reality of this object (Nake, 2012). Maser presents the Cybernetic Model of Aesthetic communication processes that deals with two main sub-processes; Aesthetic production & Aesthetic consumption. Fig. 5.

Visual Algorithmic Applications and Aesthetics

The origin of the word “Algorithm” is generally ascribed to the name of the famous Arab mathematician Muhammad ibn Musa al-Khwarizmi. The algorithm as a definition is a finite number of steps (actions), each step must be precisely defined; the actions to be carried out must be rigorously and unambiguously specified for each case (Burattini, 2013). Algorithmic visualizations are graphical tools that have been developed over the past twenty years and used intensively in different Art and Design fields. Algorithmic design is a method that creates design through algorithmic descriptions, allowing the designer to delegate repetitive tasks and accelerating the production process (Castelo-branco and Leitao, 2020).
Visual Algorithmic Design programs is a designing & analyzing algorithms tool that is used in producing technical drawings, simulating & testing, optimizing designs and implementing projects, and in generative design teaching (Khuri, 2001). To provide the ability for changing generative designs, the creators (usually programmers) have to do calculations, define the mathematical functions, geometric locations, generate shapes and forms, construct workflow, and define high order functions with sophisticated textual scripts and complicated coding languages. But with hybrid programming, designers could create generative designs with fewer programming skills. Hybrid programs provide visual components which act as generative tools that help the designer to achieve their targets and design aims, then the hybrid systems interpret these design procedures to textual code (Castelo-branco and Leitao, 2020).

Utilizing techniques of aesthetic measurements [discussed above] and developing hybrid programs (visual algorithmic programs) helped designers to produce outstanding computational aesthetics. In the next section, the paper discusses previous revolutionary generative design frameworks that have been produced through the last three centuries to redefine and develop the concept of beauty and harmony.

Development of Mathematical Aesthetics (Applied Mathematical Approach)

Mathematical Aesthetics & Birkhoff’s Aesthetic Measure

In the 1930s, George D. Birkhoff presented his general approach which appeared in his book “Aesthetic Measure | 1933” defining the objective aesthetic value that was formulated by studying and analyzing many art forms (Birkhoff, 1933). He defines an aesthetic measure (M) of an art object as the ratio between its order (O) and complexity, thus, he could interpret that physical object as numeric quantities. Then, the aesthetic measure would be a function (f) of this ratio (Douchová, 2016). The mathematical formula would be given by:

$$ M = f \left( \frac{O}{C} \right) $$

In the case of polygonal form, O will be separated into five elements of order: $O = V + E + R + HV - F$

in which, C: complexity, V: vertical symmetry, E: equilibrium, R: rotational symmetry, HV: relation to a horizontal-vertical network, F: unsatisfactory form (Birkhoff, 1933). In which the formula will be;

$$ M = f \left( \frac{V + E + R + HV - F}{C} \right) $$
Birkhoff’s applied this mathematical formula to many geometric forms Fig. 6 to provide systematic means of analysis to determine a quantitative value that defines aesthetic measurement in different forms. Birkhoff’s systematic means of analysis and its application to polygonal, ornaments and tilings had been illustrated outstandingly in his book “Aesthetic Measure” (Birkhoff, 1933).

Peirce’s Algebraic Model & Ceno-pythagorean Categories Visualization

According to Charles Sanders Peirce virtual object is not a mental copy of its real object, but a portrayal of its practical applications, predicting what and how it would produce other real objects. With this point of view, Peirce tended to attribute the source of his notion of virtuality to his own mathematical mindset, which he described as an interplay of maps and images. He presented his categories in a diagram that extends to mathematical notation and well-formed two-dimensional algebra. He uses the Pythagorean equation \( c^2 = a^2 + b^2 \) as a base for his visualized graph Fig. 7 which can be seen as a visual portrait of the relations among the variables originally standing the sides of the triangle (Danesi, 2014). Category theory, which is a branch of mathematics, could generalize all kinds of objects mathematically in terms of categories to describe the objects (variables) and their relationships as a set of morphisms.

In this context, Peirce’s Algebraic Model represented his variables as; R (the Representamen) which are entities of the real world; O (the Object) so that a determination of the mind is created, hereafter named I (the Interpretant), which establishing a triadic relation in the mind between R, O and I. This set of three


Fig. 8. 1 M. Balat’s triangular diagram for Peirce’s Ceno-pythagorean Categories. 2 M. Balat’s square diagram for Peirce’s Ceno-pythagorean Categories. 3 M. Balat’s 3D diagram for Peirce’s Ceno-pythagorean Categories. Source: Balat, Michek, (1990), Type, Trace et Ton: Le Ton Peircien, Semiosis: Internationale Zeitschrift fur Semiotik und Ästhetik.
abstract elements noted O, R, and I generates the m₁ relation between O and R, m₂ between R and I, the composed morphism m₂m₁ between O, I and the 3 identities id₀, id₁, and idᵢ represent the virtual geometrical main triangular unit in Fig. 7.1 and represented algebraically by the notion [C]:

\[
O \xrightarrow{m_1} R \xrightarrow{m_2} I \ldots \quad [C]
\]

According to Ceno-Pythagorean Categories (named; Firstness, Secondness, and Thirdness) [see above section 1.2], a new algebraic category is defined that encompasses 3 elements and 6 morphisms. The notion [Ph] is displayed in the following formula:

\[
3 \xrightarrow{i_1} 2 \xrightarrow{i_2} 1 \ldots \quad [Ph]
\]

In which arrows i₁ and i₂ are understood as logically conceiving the notion of Thirdness requires the notion of Secondness which in turn requires the idea of Firstness (Marty, 2014).

A diagrammatic method that described the principle of Peirce’s Ceno-Pythagorean categories and its ten classes of signs is shown as
Welby diagram free-hand sketches from Peirce’s manuscripts Fig. 7 (Thellefsen and Sorrensen, 2014).

In the first diagram Fig. 7.1, and according to the understanding of the universal categories [see Table 1], Peirce designed a virtual geometry with a tri-grid structure to quantities entities; in each sub-tringle, the number to the left describes; the Object of the Sign, the number in the right describes its Interpretant, and the number in the middle below describes the Sign itself. In which 1 signifies the Possible Modality, that of an Idea, 2 signifies the Actual Modality, that of an Occurrence, and 3 signifies the Necessary Modality, that of a Habit (Farias and Queiroz, 2014).

Many applications had been designed in an attempt to understand Peirce’s classifications by designing new diagrams using dimensions Fig. 8, color Fig. 9, and movement Fig. 10, aiming to explain the structural, hierarchical, and dynamic relations found in Peirce’s Cenopythagorean Categories.

Mathematical Theory of Communication & Shannon/Weaver Model

Communication, as defined by Warren Weaver (Shannon and Weaver, 1949), is a broad sense to include all the procedures by which one mind may affect another, and according to his own words; “This (communication), of course, involves not only written and oral speech, but also music, the pictorial arts, the theatre, the ballet, and in fact all human behavior."

Claude Elwood Shannon [1916–2021], Warren Weaver [1894–1978] in their book “The mathematical theory of communication | 1964" Warren Weaver mention that there are three levels of communication problems; Level A is a technical problem that is concerned with the accuracy of the sets of symbols transference from sender to receiver. Level B is a semantic problem that is concerned with the interpretation of meaning by the receiver, as compared with the intended meaning of the sender. Level C is the effectiveness problem that is concerned with the success to achieve the targeted meaning conveyed to the receiver that leads to the desired conduct. The effectiveness problem involves aesthetic considerations and it is overlapped with the semantic problem (Shannon and Weaver, 1949). While W. Weaver presented a theoretical approach to describe the communication process, C. Shannon presented a linear mathematical model that provides a framework for analyzing how messages are sent and received (Level A problem). Shannon assumes that there is information content which are transportable elements carrying messages from a given repertoire to given probabilities (Nake, 2012). If the repertoire is \( \Omega = \{ \sigma_1, \sigma_2, \ldots, \sigma_R \} \), and the probability of \( \sigma_i \) being selected is \( P_i \), where \( 0 \leq P_i \leq 1 \) and \( \sum P_i = 1 \), and \( \log \) denotes the logarithm to base 2, then the average information of a message sent by the source is:

\[
H = - \sum_{i=1}^{r} P_i \log P_i
\]

Helmar Frankn [1933–2013] and Rul Gunzenhäuser [1933–2018] developed this formula by transforming it into information-theoretic terms. Which they suggested to equate complexity with the average statistical information H, and order with the so-called relative code redundancy (Nake, 2012):

\[
R = \frac{(H_{\text{max}} - H)}{H_{\text{max}}}
\]

If all probabilities are equal when the information is maximal, then the aesthetic measure as:

\[
m = \frac{R}{H} \left( \frac{1 - H}{H_{\text{max}}} \right) = \frac{1}{H} - \frac{1}{\log r}
\]

Based on Shannon’s measure of information, in the late 1950s, Max Bense and Abraham Moles had developed new visions of informative aesthetics. Whereas Moles was interested more in sequences in music and language (time), Bense was interested in art, images, and text (space) (Nake, 2012).

Max Bense’s Information Aesthetics and Nees’s Generative Artworks

Max Bense [1919–1990] in his first book “Space and I | 1934” showed how art and mathematics are very closely connected. Many concepts had been addressed. He investigated in later books the connection between mathematics and aesthetics, in what he called the “act
of selection”. In which the act, in mathematics, was called the “selection of consciousness”, and in aesthetics, was called the “selection of feeling” (Walther, 2000). “The projects of generative aesthetics” article presents a holistic numerical aesthetics theory, that interprets artwork by mathematical logic, mathematical linguistics, and mathematical aesthetics (Bense and Nees, 1965). However, his “Contours of an Intellectual History of Mathematics | 1949” book is exclusively dedicated to the relation between mathematics and art. In this book, he introduced his Information Aesthetics theory that highlights the concept of “Mathematical Consciousness” which he considered the original action that transfers between aesthetical and mathematical categories (Walther, 2000).

Max Bense was the first to use the concept of Information Aesthetics in 1954 (Nake, 2012). His Informationsästhetik (Information Aesthetics), written in 1956, used models of communication techniques to break down aesthetic categories into aesthetic processes (Neves et al., 2014). He statistically analyzed arts to question the essence of semiotic, information theory, and communication theory using structuralistic approaches. In his point of view, the aesthetic synthesis must be preceded by analytical aesthetics to build the aesthetic structures based on the abstracted aesthetic information, which has to be described in mathematical terms (Bense and Nees, 1965). As Bense put it “the aim of generative aesthetics is the artificial production of probabilities, differing from the norm using theorems and programs.” (Caplan, 2020)

In his theoretical approach, distinguishing between Measures (material/sensual carriers like sound, color, …etc.), and Meaning (aesthetic state deduced from objects, figures, and words) in artwork, has to be done to successfully interpret the artistic phenomena. Aesthetics structure depends on specific data so-called Aesthetic Information, which falls into three main data pillars; Metrical Data that deals with composition, form, figure, and structure; Statistical Data that dealing with the concept of frequency or probability of appearance of the artwork elements; and Topological Data which presents the sets of elements in the work of art, based on notions such as environment,
connection, open state, seclusion, simplicity, and complexity. Bense mentioned that these three pillars can be manipulated and applied to an unordered set of elements to produce macro-aesthetically [complex and orderly], and micro-aesthetically [redundancies and information] (Bense and Nees, 1965).

With the inspiration of Bense’s Aesthetic Information theory, Georg Nees [1926–2016] wrote his first aesthetical programs in 1964 and finished his Ph.D. on generative aesthetics under Max Bense in 1968. Generative aesthetics produce information to bring probable alternatives of aesthetic structures by applying his aesthetic explicitly to computer art (Higgins and Kahn, 2012). In 1964, he began experimenting with what he called “statistical graphics” by accessing [Zuse Graphomat Z64], a programmable drawing machine (Caplan, 2020). By using a simple algorithm to instruct the machine and with aesthetically balanced forms, he was able to produce 2D printed artworks. Nees described his experiment in Fig. 11; “… programs of stochastic computer graphics a drawing table controlled by erasure strips and a digital calculator were used to produce the graphics. Every graphic has random parameters, the program repeats generating basic operations for the individual graphic in such a way that the mere repetitions create the aesthetic redundancy, and the random parameter values with each repetition create the aesthetic improbability of the graphics.” (Bense and Walther, 1968).

Stakhov’s Mathematics of Harmony

Ukrainian scientist Professor Alexey Stakhov [1939–2021] since his first book, “Introduction into Algorithmic Measurement Theory | 1977”, and his “Hyperbolic Fibonacci and Lucas Functions book | 2003”, introduced a new mathematical direction so-called the “Algorithmic Measurement Theory”, focused on stimulating research in the field of theory depending on Fibonacci numbers and the Golden Section (Stakhov, 2009). In 2004, he generalized the Golden Section formula by setting a non-negative integer \( p = 0, 1, 2, 3, \ldots \) and divided a line segment \( AB \) at the point \( C \) in the following proportion:

\[
\frac{CB}{AC} = \left( \frac{AB}{CB} \right)^p
\]

which can be transformed into the following algebraic equation:

\[
x^{p+1} = x^p + 1
\]

This equation which is called Golden \( p \)-proportions generalized the traditional golden proportion. And building on the golden \( p \)-proportion Stakhov introduced a new formula of real numbers;

\[
A = \sum_i a_r^i, \ a_i \in \{0, 1\}
\]

which presented a new arithmetic system that was used for so-called Fibonacci computers. (Stakhov, 2009);

“The Fibonacci computer uses the Fibonacci numbers as the number base in which calculations and operations are performed with the fault tolerance coming from the redundancy present in this base. this context completeness means that any nonnegative integer can be expressed in terms of the base numbers with appropriate coefficients.” (Ligomenides and Newcomb, 1981).

Nature-inspired Algorithms and Arithmetic Aesthetics

Studying brain, cognition, and visual system in humans that measures or quantifies aesthetics and trying to imitate methods of human aesthetic measurements are great challenges. Mimicking these methods to create a well-designed product is also a challenge. Many possible aesthetic criteria and evaluation methods could be used to achieve beauty and harmony in design. The algorithmic design method presents a wide design space for designers to achieve their goals (Castelo-Branco, 2020). Fig. 12 presents an example of an Algorithmic Design for creating a ‘Phyllotaxis Pattern’, an arithmetic form that would be difficult to model by using traditional design methods. Phyllotaxis geometry considering the ubiquity of the Fibonacci numbers in nature Fig. 12.1. The design criteria in Fig. 12.2 used a visual algorithmic program and two simple pattern algorithms that are inspired by nature;
Algorithmic Generative Design and Mathematical Harmony (implementation approach)

The designs in Fig. 14 aim merely to copy the natural arrangements of the form and to adapt it to create alternative possible design solutions for the generative design by applying mathematical notations based on the Fibonacci sequence algorithmic function.
The points \((X, Y)\) at the coordinate system were defined in the following formula:

\[
X = \cos(t) \times t
\]

\[
Y = \sin(t) \times t
\]

In which \((t)\) is the defined numeric domain. Also, the Voronoi component (nature-inspired algorithm) was applied to some designs.

Design patterns in Fig. 14 were generated from a model inspired by the Fibonacci generative rule in nature. The algorithmic experiments are based on biomimetic reference; Phyllotaxis and had been adapted and reparameterized to generate alternative design solutions. Understanding the hierarchical level of harmony, studying the mathematical

Fig. 14. Parametric Generative designs Phyllotaxis by using two simple pattern algorithms that are inspired by nature; Voronoi diagram-based algorithm and the Fibonacci sequence algorithm. (Visual algorithmic programs; Rhinoceros/Grasshopper). Source: Naturally Generative Algorithmic Designs by Nessreen Y. Ibrahim (Author).
equations, and applying naturally inspired notation, and using Algorithmic Aesthetics generative methods provide solid references for designers to apply the aesthetic values in their produced work and help them to generate several well-designed parametric design variants.

Conclusion and Future Work

The history of aesthetics as a philosophical branch of knowledge presented many theoretical and applied approaches that investigate the concept of mathematical harmony. Examples were discussed in this paper as an attempt to emphasize the important role of mathematical notations and algorithmic functions in developing a new aesthetic theory. The algorithmic aesthetic is a challenging intellectual theory that goes beyond the traditional aesthetics domains. It is expected that the near future will combine the digital-computational-informational-intelligent applications with the bio-neuro-cognitive aspects to generate a revolutionary scientific-united era. This article presented a holistic review of the concept of mathematical harmony and its ingrained theories throughout the written history from Greek philosophers to the 1900s scientists and artists, as a primitive study aiming to embody this future vision in the field of aesthetics. In the context of algorithmic constructivism, the research presented a primitive attempt to generate parametric designs that act as alternative patterns, that were all based on a nature-inspired algorithm.

The research paper is a review of previous literature in the field of research, where the future research work intended to develop a theory that would allow one model to measure the amount of information flow in any aesthetic phenomenon, thus the main target will be to define an aesthetic algorithm that will be able to interpret the quantity and quality of aesthetic values as numerical information.

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